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Properties of the collisional plasma sheath with ionizations

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Abstract. A simple model of a collisional plasma sheath, with ionizing collisions included and in contact with a negative metal wall, is proposed in this paper. A broad range of plasma sheath solutions is obtained and their properties are discussed. The characteristic of the abnormal glow discharge as well as the characteristic of the electrostatic probe, in the ion-saturation region, are incidentally recovered.

This theory is then applied to the study of current multiplication in electrical discharges. The properties of a 'plasma current multiplier' are predicted, both in the low and high pressure regimes, and are found to be in general agreement with the observed operating characteristics reported elsewhere.

1. Introduction

In this paper we propose a comparatively simple model of a collisional plasma sheath, with ionizations included, in contact with a negative metal wall. We have in mind ionized gases at gas pressures of the order of 100 mTorr upwards and wall bias of a few tens up to a few hundred volts. In these physical circumstances the motion of the charge carriers across the sheath is mobility controlled, secondary emission of electrons by ion bombardment at the wall is relevant and these electrons, streaming in the high field prevailing in the sheath, produce charge carrier multiplication.

The self-consistency of the model of a collisional plasma sheath requires the computed sheath thickness x_1 to be far larger than the electron and ion mean free paths (l^- and l^+). That is to say, the condition $x_1 p \gg lp\,(l = l^-$ and $l^+)$ must be verified. Noticing that $P = 1/lp$ is the effective collision cross section or probability of collision the required condition can be put in the form $x_1 p \gg 1/P\,(P = P^-$ and $P^+)$. In most gases the actual values of P for both electron–neutral elastic collisions and ion–neutral charge-exchange collisions (for ions in their parent gas) is far above $10^3\ \mathrm{m^{-1}\,Torr^{-1}}$ over most of the permissible particle energy range (up to a few hundred electron volts). The present model will accordingly be generally applicable when the computed sheath thickness satisfies the criterion $x_1 p > 0{\cdot}1 \times 10^{-2}$ m Torr (metre torr). In the case taken as an example and for which results are reported in this paper this criterion is verified over a broad range of sheath solutions.

The secondary electrons, released into the sheath by ion bombardment on the wall, produce charge multiplication when drifting in the high electric field of the sheath. This is a mechanism of current multiplication upon which the concept of 'sheath current gain' is based. Its dependence on the wall bias, collected current density and gas pressure is investigated. The electric potential distribution in the sheath is also obtained as a function of the same parameters.

The present theory is then applied to the limit case of the self-sustained (infinite current gain) plasma sheath; the cathode fall of the glow discharge and the characteristic of the abnormal glow discharge are found in this way. The theory is next applied to the volt–ampere characteristic of the electrostatic probe in its ion saturation regime.

In § 3 of this paper we proceed to study current multiplication in electrical discharges. A 'plasma current multiplier' operating at low pressure was first described by Stangeby and Allen (1971a). The mechanism of current multiplication originally proposed by those authors is based on volume ionization followed by charge carriers separation thus giving rise to discharge current build up.

We now propose a complementary mechanism, based on ionization and current multiplication in the sheath, to be taken into account at high operating pressure. With the help of this mechanism one can further the understanding of the 'plasma current multiplier' and explore its potential capabilities at higher pressures.

2. Theory of collision dominated ion sheaths with ionizations

2.1. *Basic theory*

Let us consider a collision dominated, one-dimensional (plane), plasma sheath in contact with a negatively-biased metal wall. Assume that the relevant mechanisms in the sheath and at its boundaries are the following:

(i) a positive ion current enters the sheath at the sheath–plasma interface;

(ii) an electron current, induced by ion bombardment, is emitted by the metal wall;

(iii) in the volume, the electron current drifting in the applied electric field gives rise to charge multiplication.

Take an x axis perpendicular to the plane of the sheath and place the origin of coordinates at the sheath–plasma interface. Let i_0^+ and i_1^+ (i_0^- and i_1^-) denote the positive ion (electron) current densities at the plasma and wall boundaries of the sheath respectively. This situation is schematically represented in figure 1.

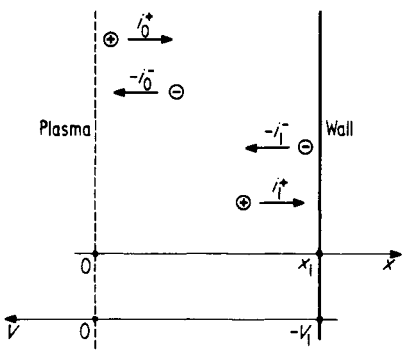

Figure 1. The plasma sheath next to a negatively biased wall.

The total current density is given by

$$j(x) = i^+(x)+i^-(x) = i_1^+ + i_1^- = i_0^+ + i_0^- . \tag{1}$$

Let γ^+ be the secondary-emission coefficient at the wall, that is to say, the number of electrons ejected per incident ion:

$$\gamma^+ = \frac{i_1^-}{i_0^+ + i_0^- - i_1^-}. \tag{2}$$

This is supposed to be a constant with regard to the applied voltage, current and gas pressure over the range of parameters covered in the computations reported in the present paper.

Charge multiplication in the sheath, produced by ionizing collisions of the electrons with the gas atoms, is governed by an exponential law that involves the first Townsend coefficient α:

$$i^-(x) = i_1^- \exp\int_x^{x_1} \alpha(x)\,\mathrm{d}x.$$

For the E/p range (the electric field to gas pressure ratio) that prevails across most of the sheath one may take the following approximation for α (von Engel 1965):

$$\alpha = Ap\exp\left(-\frac{Bp}{E}\right)$$

so that

$$i^-(x) = i_1^- \exp\int_{-V_1}^{V} \frac{Ap}{E}\exp\left(-\frac{Bp}{E}\right)\mathrm{d}V. \tag{3}$$

From equations (1) to (3) one can deduce the local ion current density

$$i^+(V) = j\left[1 - \frac{\gamma^+}{1+\gamma^+}\exp\int_{-V_1}^{V} \frac{Ap}{E}\exp\left(-\frac{Bp}{E}\right)\mathrm{d}V\right]. \tag{4}$$

The motion of the charge carriers across the sheath is determined by the local field and this, in turn, is governed by Poisson's equation

$$\frac{\mathrm{d}^2V}{\mathrm{d}x^2} = -\frac{e}{\epsilon_0}(n^+ - n^-) \tag{5}$$

where n^+, n^- are the particle number densities. In view of the large bias voltages we are interested in, the negative bias voltage of the wall relative to the plasma is always far higher than the electron and ion temperatures in volts, so that the electron number density is negligibly small within the sheath and the ion motion is mobility dominated:

$$n^- = 0$$
$$n^+ = \frac{i^+}{e\mu^+E} \tag{6}$$

where μ^+ denotes the positive ion mobility.

Under the same assumption, moreover, the electric field at the plasma boundary will always be negligibly small as compared to the field inside the sheath, and so one may write

$$E = 0 \qquad (x = 0)$$

or, taking as reference the potential at the plasma boundary,

$$E = 0 \qquad (V = 0). \tag{7}$$

Noting that $d^2V/dx^2 = E\,dE/dV$ and $E = -dV/dx$, one can now write, after equations (5) and (6),

$$E^2\frac{dE}{dV} = -\frac{i^+}{\epsilon_0\mu^+}.$$

Taking equation (4) into account and introducing the similarity quantities E/p and j/p^2, one finally arrives at the following differential equation:

$$\left(\frac{E}{p}\right)^2\frac{d(E/p)}{dV} = -\frac{j/p^2}{\epsilon_0\mu^+p}\left[1-\frac{\gamma^+}{1+\gamma^+}\exp\int_{-V_1}^{V}\frac{Ap}{E}\exp\left(-\frac{Bp}{E}\right)dV\right] \tag{8}$$

which is subjected to the boundary condition (7). Note that μ^+p is a very slow function of E/p up to moderately large fields (Brown 1959, chap 3); one may accordingly assume this to be a constant in order to facilitate the integration procedure.

Once the gas and the metal wall are specified, one is left simply with the three variables $E/p, j/p^2$ and V. Equation (8) is to be integrated in V starting from the wall at $-V_1$, with a tentative initial value E_1/p, towards the plasma boundary; the initial value E_1/p was then varied and adjusted until the boundary condition (7) was satisfied. In this way one may obtain the sheath solution for any specified pair of values of the applied voltage V_1 and the drawn current j/p^2.

The most relevant quantity obtained from each solution, and the one upon which much of the forthcoming analysis is based, is

$$g = \frac{j}{i_0^+} = \left[1-\frac{\gamma^+}{1+\gamma^+}\exp\int_{-V_1}^{0}\frac{Ap}{E}\exp\left(-\frac{Bp}{E}\right)dV\right]^{-1}. \tag{9}$$

g may be named 'the sheath current gain' for it is the factor by which the ion current leaving the plasma is multiplied when passing across the sheath.

2.2. *Some numerical results*

2.2.1. General features of the sheath solution. The computational results reported here were obtained with the PDP-15 computer of this laboratory using the following values for the gas parameters and secondary emission coefficient: $A = 1{\cdot}36\times10^3\ \text{m}^{-1}\,\text{Torr}^{-1}$, $B = 2{\cdot}35\times10^4\ \text{V}\,\text{m}^{-1}\,\text{Torr}^{-1}$ and $\gamma^+ = 0{\cdot}100$. The values assumed for these quantities are not claimed to apply to a particular gas and wall material but are all physically reasonable. The values of A and B correspond, however, to early estimates relative to argon due to Townsend (Cobine 1958, chap 7) and the value of γ^+ may be taken as applying to argon ions on atomically clean tungsten (Brown 1959, chap 11).

SI units, with the exception of the unit of pressure (torr, for practical convenience) are used.

The values obtained for the sheath current gain, at constant values of the collected current, are plotted in figure 2 against the applied voltage. Notice that g is always a monotonically increasing function of V_1. At higher j/p^2 g grows rapidly to infinity at finite V_1. This behaviour is interpreted as referring to the approach to the self-sustained cathode sheath of a glow discharge. The minimum values of j/p^2 and V_1 for which the self-sustained sheath occurs is interpreted as referring to the normal cathode of the glow discharge (found at about $(j/p^2)/(\epsilon_0\mu^+p) = 0{\cdot}5\times10^{12}\ \text{V}^2\,\text{m}^{-3}\,\text{Torr}^{-3}$ and $V_1 = 155$ V).

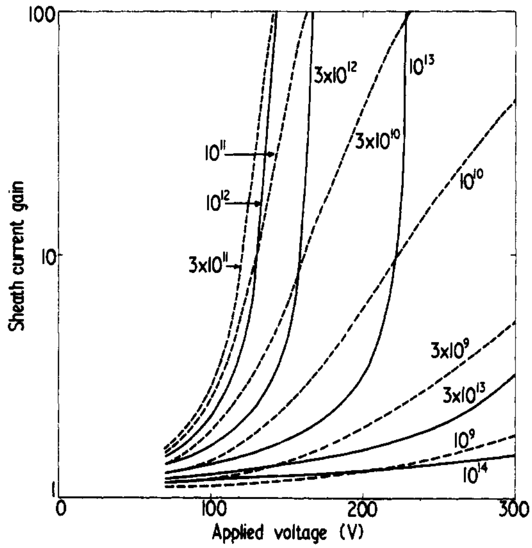

Figure 2. Dependence of the sheath current gain on the applied voltage for different values of the collected current density. The plotted curves are labelled in terms of $(j/p^2)/(\epsilon_0\mu^+p)$.

In tables 1 and 2 one has kept a record of x_1p and E_1/p obtained from a broad range of solutions, in order to have an idea of the ranges covered by the variation of those parameters.

Table 1. Values of $x_1p(10^{-2}$ m Torr)

V_1(V)	$(j/p^2)/(\epsilon_0\mu^+p)(V^2\,m^{-3}\,Torr^{-3})$				
	10^{10}	10^{11}	10^{12}	10^{13}	10^{14}
50	0·68	0·32	0·149	0·068	0·031
100	1·14	0·60	0·261	0·111	0·050
150	1·69	1·38	0·825	0·151	0·066
200	2·57	3·26	—	0·200	0·081
250	3·90	—	—	—	0·095
300	5·68	—	—	—	0·109

Table 2. Values of $E_1/p(10^2\,V\,m^{-1}\,Torr^{-1})$

V_1(V)	$(j/p^2)/(\epsilon_0\mu^+p)(V^2\,m^{-3}\,Torr^{-3})$				
	10^{10}	10^{11}	10^{12}	10^{13}	10^{14}
50	110	234	503	1091	2376
100	134	276	611	1363	2980
150	144	282	638	1536	3396
200	147	282	—	1650	3720
250	148	—	—	—	3988
300	148	—	—	—	4215

At low applied voltages, the electric field within the sheath varies as $E \propto x^{1/2}$ or $-V \propto x^{3/2}$ (which corresponds to the distribution of the electric potential in a collisional sheath without ionizations, in the case when the frequency of collisions is independent of the particle speed, see Cobine 1958, chap 6). As the applied voltage is increased, the electric field variation approaches a $E \propto x$ or $-V \propto x^2$ law (this distribution of the electric field was observed in the cathode fall of glow discharges, see Stein 1953) provided that the collected current density is comparable to or above that of the normal cathode of the glow discharge. However, if the applied voltage is increased further, the electric field is found to follow even faster laws of variation as the self-sustained cathode sheath is approached. These facts are illustrated in the plots of figure 3.

The results obtained at higher voltages for the electric potential distribution (see figure 3) and sheath parameters (see tables 1 and 2) point to the identification of two regions in the sheath: the sheath proper, where the field is high, and the pre-sheath, where the field is low, but that may cover much of the sheath thickness and voltage drop.

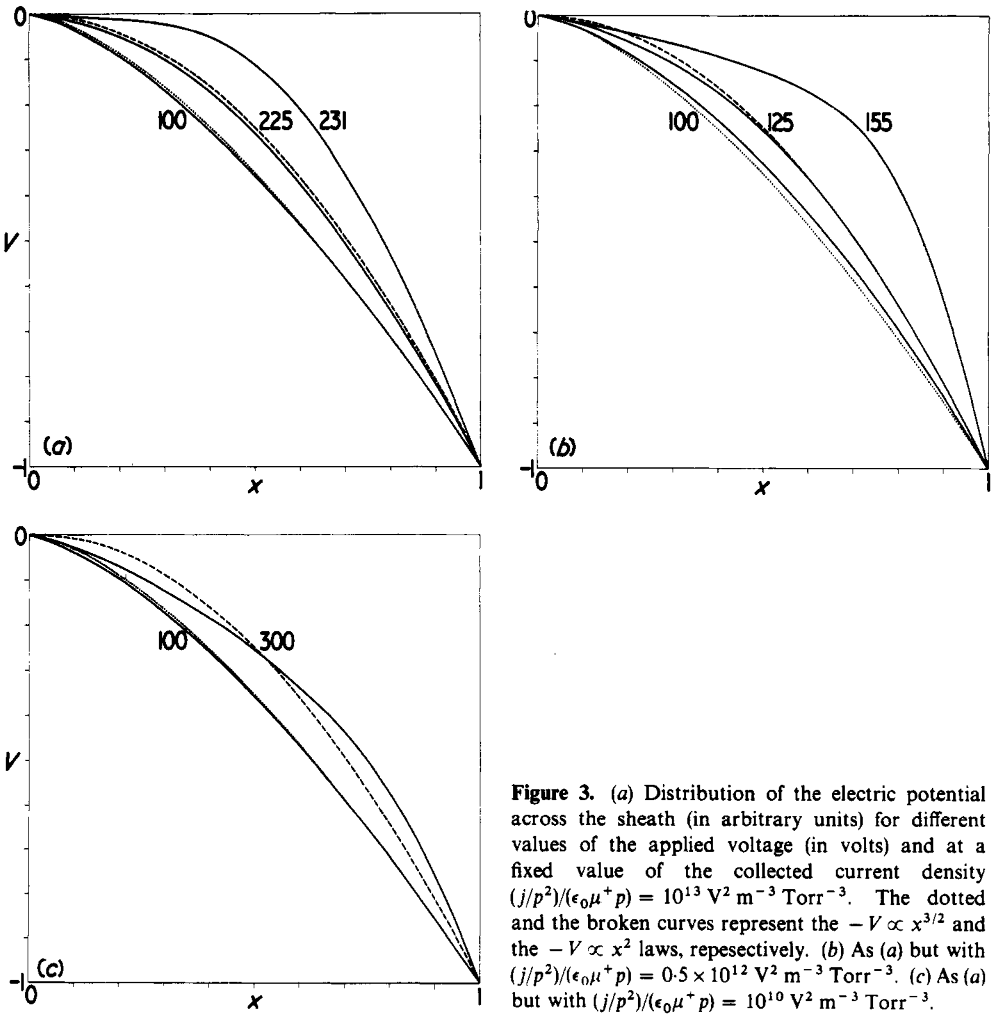

Figure 3. (*a*) Distribution of the electric potential across the sheath (in arbitrary units) for different values of the applied voltage (in volts) and at a fixed value of the collected current density $(j/p^2)/(\epsilon_0\mu^+ p) = 10^{13}\ \mathrm{V^2\ m^{-3}\ Torr^{-3}}$. The dotted and the broken curves represent the $-V \propto x^{3/2}$ and the $-V \propto x^2$ laws, repesectively. (*b*) As (*a*) but with $(j/p^2)/(\epsilon_0\mu^+ p) = 0{\cdot}5 \times 10^{12}\ \mathrm{V^2\ m^{-3}\ Torr^{-3}}$. (*c*) As (*a*) but with $(j/p^2)/(\epsilon_0\mu^+ p) = 10^{10}\ \mathrm{V^2\ m^{-3}\ Torr^{-3}}$.

2.2.2. The characteristic of the abnormal glow. For cathode current densities above that of the normal glow discharge, the sheath current gain is generally decreasing with the collected current, at a fixed applied voltage, as shown in figure 4. Notice that g grows to infinity at a finite value of j/p^2, which is higher the higher the value of V_1. Figure 5 shows the relation between the values of j/p^2 and V_1 for which self-sustained sheaths are found ($g = \infty$); this curve is interpreted as the characteristic of the abnormal glow discharge.

2.2.3. The ion saturation regime with ionization. In figure 6 one finds the plot of the collected current against the applied voltage at a number of fixed values of the positive

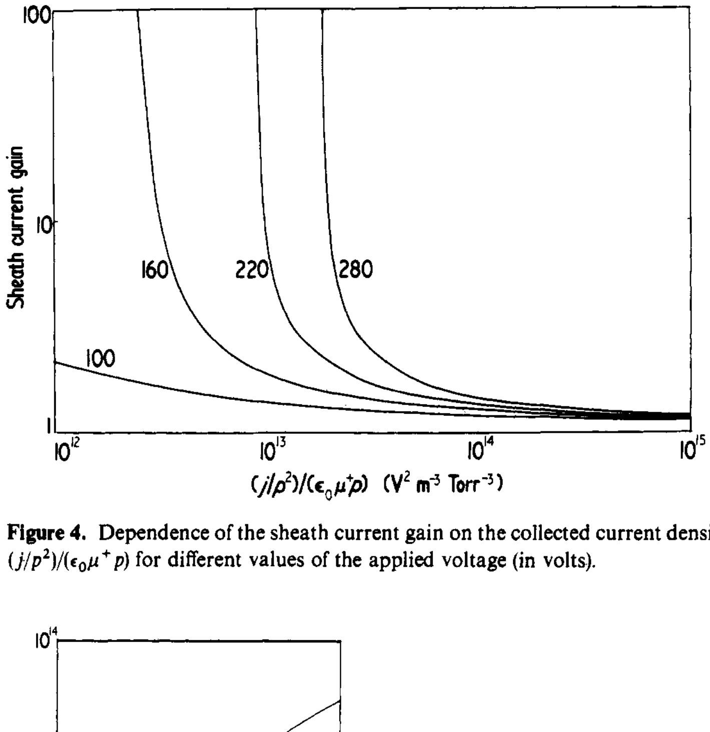

Figure 4. Dependence of the sheath current gain on the collected current density in terms of $(j/p^2)/(\epsilon_0\mu^+p)$ for different values of the applied voltage (in volts).

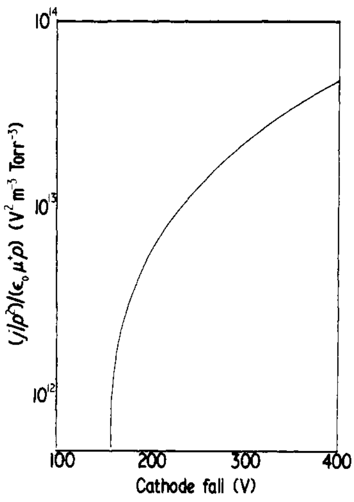

Figure 5. Abnormal glow-discharge characteristic. The cathode current density is expressed in terms of $(j/p^2)/(\epsilon_0\mu^+p)$.

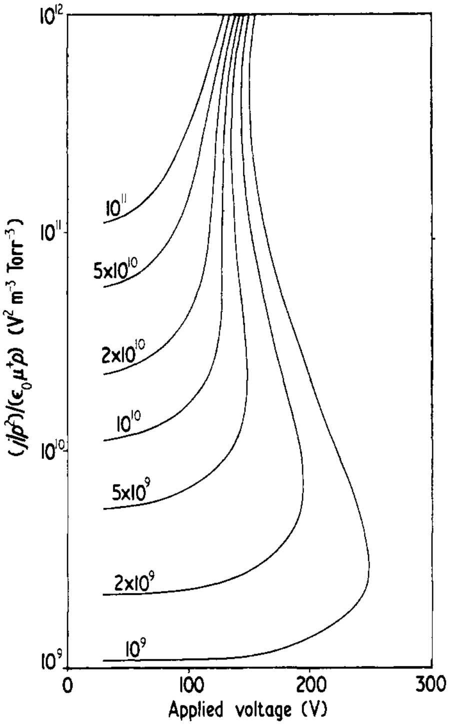

Figure 6. Electrostatic probe characteristics in the ion saturation regime, with ionization included, for different values of the positive ion current density entering the sheath. The plotted curves are labelled in terms of $(i_0^+/p^2)/(\epsilon_0\mu^+p)$ and the collected current density is expressed in terms of $(j/p^2)/(\epsilon_0\mu^+p)$.

ion current entering the sheath (that is, at constant $i_0^+ = j/g$). The curves thus obtained are interpreted as electrostatic probe characteristics in the ion saturation regime with ionizations included. Notice how the collected current increases by orders of magnitude as the applied voltage is increased. Notice also how the characteristic falls into a negative resistance regime, for the more tenuous plasmas, going through a minimum at values of the current density and voltage which are comparable to those of the cathode of the normal glow discharge, thus suggesting the onset of an autonomous glow.

3. Application of the above theory to current multipliers

3.1. Electrical discharges as current multiplication devices

In a number of recent papers, the principle and some properties of a 'plasma current multiplier' have been described (Stangeby and Allen 1971a, 1972, Stangeby and Rosa 1972a, b).

Consider a glow discharge running within a metal-walled discharge tube. When the wall is biased negative relative to the floating potential the anode current is larger than the cathode emission current. Charge carriers produced by volume ionization are separated, the ions being collected by the negatively-biased wall and the electrons adding up to the axial current arriving at the anode. This current multiplication mechanism was originally proposed by Stangeby and Allen (1971a).

We now believe that a complementary mechanism, based on current multiplication in the sheath, ought to be taken into account as well at high operating pressures. With the help of this mechanism one can further the understanding of the 'plasma current multiplier' and anticipate arbitrarily large gains per unit length at finite discharge currents.

3.2. *Theory of the plasma current multiplier*

Let $I(z)$ denote the axial discharge current and $j(z)$ the current density collected by the wall. In a cylindrical discharge tube of radius r current gain is related to the ion wall current:

$$\frac{\mathrm{d}I}{\mathrm{d}z} = 2\pi r j = 2\pi r i_0^+ g,$$

g being the 'sheath current gain' introduced in § 1 of this paper. Let us now define λ, the ratio of the ion current leaving the plasma column per unit length to the axial discharge current:

$$\lambda = \frac{2\pi r i_0^+}{I}.$$

Combining these two equations, one arrives at the following current gain per unit length

$$\frac{\mathrm{d}I/\mathrm{d}z}{I} = \lambda g$$

so that current build up along the axis is given by

$$I = I(0)\exp\int_0^z \lambda g \,\mathrm{d}z$$

and the overall anode to cathode gain of the 'plasma current multiplier' is

$$G = \frac{I}{I(0)} = \exp\int_0^z \lambda g \,\mathrm{d}z.$$

If λg is roughly constant along z then $G = \exp(\lambda g z)$, that is, the gain of the current multiplier (i) varies exponentially with its length; this has generally been observed to be the case.

3.2.1. *The low pressure regime.*

Current gain per unit length is given by the product λg. If there is no ionization in the sheath, g is nearly constant and close to unity ($g = 1+\gamma^+$). Current multiplication is then associated with volume ionization alone and its magnitude determined by λ, as originally proposed by Stangeby and Allen (1971a).

When ionization in the sheath is negligible, the ion wall current is proportional to the axial discharge current (see Stangeby and Allen 1971b), in which case λ is given by

$$\lambda = \left(\frac{\alpha}{\beta\gamma}\right)\left(\frac{2\pi m^-}{m^+}\right)^{1/2}\frac{2}{r}$$

where α represents the effect of velocity dispersion on the ion wall current density, β is the ratio of drift to random electron current and γ is the ratio of the average number density across the column to the number density at the plasma boundary.

We remember how the radius of the plasma column is affected by the plasma density and wall bias and invoke the $1/r$ dependence of λ, then it can be predicted (Stangeby and Allen 1972, private communication) that the gain of the current multiplier (ii) will grow with the biasing voltage; (iii) will decrease with the collected current.

If we allow λ to vary along z, as λ decreases down to an asymptotical value with j (as r approaches the discharge tube radius) and j grows monotonicly with z, one can predict that the gain of the current multiplier (iv) is faster than exponential at shorter distances before settling down at longer ones.

Arbitrarily large multiplier gains per unit length would be possible ($\lambda \propto 1/r$) but at the cost of vanishingly small discharge currents ($I \propto r^2$).

3.2.2. The high pressure regime. At high pressure, current multiplication will be mainly associated with ionization in the sheath and its magnitude determined by g.

Recalling the general behaviour of g with the discharge parameters as shown in figures 2 and 4 of § 2 of this paper, it can be predicted that properties (ii), (iii) and (iv), found in the low pressure regime, will also hold true at high pressure although by means of a different mechanism. However, one can further predict that the gain of the current multiplier (v) will grow with the gas pressure.

If ionization in the sheath is allowed, g may grow by several orders of magnitude and so will the multiplier gain, at finite discharge currents; this is another new feature of the high pressure regime.

3.3. Conclusions

Both volume and sheath ionization will, in general, contribute to the overall gain of the 'plasma current multiplier'. One or the other will be predominant according to the pressure range at which the discharge is operated.

The exponential growth of the multiplier gain with its length (property referred to as (i) in the preceding section) was generally observed (Stangeby and Allen 1971a, 1972, Stangeby and Rosa 1972b). A current growth faster than exponential at the initial stages of the multiplier (property (iv)), already hinted at by earlier results, was now made apparent and emphasized by the work of Stangeby and Allen (1972, private communication).

The observed operating characteristics of a 'plasma current multiplier' (Stangeby and Rosa 1972b) are in qualitative agreement with properties (i) to (iii) predicted in the preceding section. In the course of their experiments the same authors also observed the growth of the multiplier gain with gas pressure (property (v)), this is apparently specific to the high pressure regime, although they were still operating in an intermediate pressure range (from 50 to 160 mTorr).

Arbitrarily large multiplier gains at finite discharge currents are anticipated, in the high pressure regime, provided that the discharge parameters are adjusted to attain sufficiently high values of the 'sheath current gain'. In the limit, when the 'sheath current gain' becomes infinite, control is lost of the initial cathode emission current, the device no longer behaves as a current multiplier and the metal-walled discharge tube turns into a hollow cathode.

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